Problems Section

Optimal Measuring Signal Generation*

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Motivation and General Form of the Problem

Respiratory mechanics is commonly studied by means of external driving signals applied to the measured system [5]. These signals are preferably of composite waveforms, i.e., they contain many components of different frequencies within a repetition period T. Such a "pseudorandom" signal U can be written in the form of a finite Fourier series [4]:

$$U(t, T, M, D, F) = \sum_{k=1}^{L} d(m_k) \cos\left[\frac{2\pi m_k t}{T} + \phi(m_k)\right], \tag{1}$$

where $M = [m_1, \ldots, m_L]^T$ is the component number vector ($[.]^T$ denotes the transposed vector), m_k-s are positive integers, $D = [d(m_1), \ldots, d(m_L]^T$ is the component amplitude vector ($d(m_k) > 0$), and $F = [\phi(m_1), \ldots, \phi(m_L)]^T$ is the component phase vector. Since the measured system is generally nonlinear, the overall size of the signal should be kept as low as possible, but with sufficient power at each frequency component to attain acceptable signal-to-noise ratios. In other words, the problem is to find a vector F for prescribed T, M and D such that the function

$$A(F) = \frac{1}{\Sigma_{k=1}^{L} d(m_k)} \max_{0 \leq t < T} | U(t, T, M, D, F) | \tag{2}$$

be minimal subject to the bounds $0 \leq \phi(m_k) \leq 2\pi$. Similar optimization problems arise with somewhat different objective functions [1, 2, 6, 7].

Problem (2) may have many local minima, and hence it is a global optimization problem. It is easy to see that (2) has a maximum at $F = 0^T$ with $A = 1$(pessimum signal). The optimal signal may be shifted with the same objective function value, and thus one variable may be fixed (e.g. $\phi(m_1) = 0$). Obviously, transformations such as $U' = -U$ and $\phi'(m_k) = 2\pi - \phi(m_k)$ do not change $A(F)$.

Particular Case

Consider problem (2) with $L = 64$, $T = 1$, and $d(m_k) = 1$ for $m_k = 4k$, $k =$

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1, . . . , 64. Because of the way the signal is applied, it is sufficient to calculate the maximum of U on a discrete set of t values (e.g. $t_j = jT/N$, for $j = 0, \ldots, N-1$). Set $N = 256$.

The optimizations were carried out with random walk and clustering global optimization methods [3] on an IBM AT compatible computer, and a local minimization needed on average half an hour of CPU time. The best local minimum found after some hundred local searches was 0.15730166. No local minimum is unique according to the transformations shown in the first section, and thus we do not give here the best optimizer point found (it will be sent on request to those interested). The difficulty of the problem is characterized by the fact that we did not see any given local minimizer point twice.

It is still an open question as to whether it is possible to give reasonable lower and upper bounds for the global minimum of the general problem, or how one can find approximate solutions efficiently, or which parameter transformations reflect the redundant structure of (2).

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